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# MODELLING OF THE PROCESSES INSIDE THE NOZZLE OF RESISTOJET AND ARCJET WITH MOLECULAR PROPELLANT

*The first sample of the resistojet, which together with the arcjet belongs to the class of electrothermal thrusters, was developed in 1929-33 by the Ukrainian designer Valentyn Hlushko. Electrothermal thrusters combine the gas-thermodynamic principle of acceleration in a profiled nozzle with the use of electrical energy to heat the propellant. In resistojets, energy is supplied to the gas by an ohmic heater. In arcjets, the source of energy is the arc discharge. The advantage of this type of thruster is the greater value of the specific impulse compared to chemical rocket engines with solid or liquid propellants. In view of the inverse proportionality of the specific impulse to the square root of the molecular weight, hydrogen was initially chosen as the propellant. However, the operation of electric propulsion thrusters involves long-term propellant storage and excludes the use of hydrogen in liquid form. When stored in a gaseous state, due to the significant rarefaction of hydrogen, the mass of the balloon significantly exceeds the mass of hydrogen itself, negating the gain in propellant mass reduction compared to chemical engines. Interest in the development of electrothermal thrusters was revived by the proposal to use molecular propellants: ammonia and hydrazine. A mathematical model and a method for calculating thermochemical processes in the primary dissociation and secondary synthesis reactions of molecular nitrogen and hydrogen in the nozzle of an electrothermal thruster were developed in this work. The calculation method involves the use of continuity, momentum, and energy equations, considering the change in the molecular composition as the gas expands in the nozzle. The problem of lack of equilibrium constants was solved by generalizing the Saha ionization formula to the case of the interaction between nitrogen and hydrogen atoms. The calculation results demonstrate the possibility of obtaining a specific impulse at the level achieved when working on hydrogen. The necessity of using the non-stationary form of the equations during the creation of the process model in the heater or thruster arc is also indicated.*

***Keywords****: arcjet; resistojet; Saha ionization formula; dissociation; synthesis; nozzle; ammonia; hydrazine.*

## Introduction

Electrothermal thrusters (ETT) are the simplest class of electric propulsion thrusters (EPT) and combine the thermodynamic principle of acceleration, as in chemical rocket engines, with the supply of heat to the propellant by converting electrical energy into thermal one. ETT include two main types: resistojet (RJ) with heating of the propellant from an ohmic heater and arcjet (AJ) with heating of the propellant in an electric arc.

The first sample of RJ – the ERD-1 thruster, developed in 1929-1930 under the leadership of the outstanding Ukrainian designer Valentin Glushko [1].

The use of solid heater makes a limit operating about 2700 K. So maximum specific impulses are reached around 3000 m/s. The highest specific impulse is obtained by using hydrogen as propellant [2]. The long-term operation of EPT excludes the possibility of storing hydrogen in a liquid state at temperatures below 20 K. When stored in a gaseous state, the volume of the tank is large, and the mass of the balloon is several times greater than the mass of the hydrogen contained in it.

The storage in liquid state is possible with use of such molecular propellants as ammonia and hydrazine.

However, with the gradual development of ion thrusters and Hall thrusters, ETT are gradually being replaced. However, since the hydrazine ETT can share the hydrazine in the tank with the hydrazine LPRE, this combination makes the structure of the orbit control system relatively simple (the hydrazine LPRE is used for attitude control, and the hydrazine ETT is used for position maintenance). Therefore, the combined use of hydrazine LPRE and hydrazine ETT still has good research and application prospects in China [3].

The purpose of this work is to develop a mathematical model and methodology for calculating the characteristics of the gas flow in the ETT nozzle when operating on ammonia or hydrazine, taking into account changes in the component composition with changes in gas temperature and pressure.

## Statement of the research problem

The stationary adiabatic flow of a molecular gas mixture in the ETT channel should be described by continuity, momentum and energy equations:

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$$\rho_M V S = \dot{m}; \quad (1)$$

$$\rho_M V \frac{dV}{dx} + \frac{d(nkT)}{dx} = 0; \quad (2)$$

$$\frac{V^2}{2} + \varepsilon_M = \varepsilon_M^{(0)}, \quad (3)$$

where $\rho_M$ – mass density; $V$ – mass flux velocity; $T$ – temperature; $n$ – molecular population; $S$ – channel cross-section; $\varepsilon_M$ – the internal energy of a mass unit; $\rho_M V S = \dot{m}$ – mass flow rate; $k$ – Boltzmann's constant.

The total characteristics of the mixture were calculated based on the partial volume fractions $s_\alpha$:

$$\rho_M = n\sum_\alpha m_\alpha s_\alpha, \quad (4)$$

$$\varepsilon_M = \frac{\sum_\alpha \left(C_{P\alpha}T + \varepsilon_\alpha\right)s_\alpha}{\sum_\alpha m_\alpha s_\alpha}, \quad (5)$$

where $\varepsilon$ – the state energy.

Only the momentum equation requires the use of numerical solution methods.

The primary dissociation reactions of $NH_3$ and $N_2H_4$ correspond to the dissociation energy values in electron volts given below in parentheses: $NH_3 \rightarrow NH_2+H$ (4.48), $NH_2 \rightarrow NH+H$ (3.90), $N_2H_4 \rightarrow N_2H_3+H$ (2.60), $N_2H_3 \rightarrow N_2H_2+H$ (3.05), $N_2H_2 \rightarrow 2\,NH$ (4.50), $NH \rightarrow N+H$ (3.50). The secondary reactions of synthesis of $N_2$ and $H_2$ correspond to the values of energy released during synthesis given below in parentheses: $2\,H \rightarrow H_2$ (4.48), $2\,N \rightarrow N_2$ (9.76) [4].

Under the assumption of thermodynamic equilibrium, the molecular composition could be calculated taking into account the concept of the equilibrium constant of a chemical reaction. However, there is a lack of experimental data for the equilibrium constants of the entire variety presented above. In our calculations, we used the Saha formula [5] with the replacement of the electron mass by the reduced mass of the reaction products and ionization energy by the dissociation energy:

$$\frac{s_\alpha s_\beta}{s_\gamma} = \frac{2}{n}\left(\frac{m_p kT}{2\pi\hbar^2}\frac{\mu_\alpha\mu_\beta}{\mu_\gamma}\right)^{3/2} e^{-\frac{\varepsilon_\alpha+\varepsilon_\beta-\varepsilon_\gamma}{kT}}, \quad (6)$$

where $\mu$ – the molecular weights of the components in atomic mass units; $m_p$ – the mass of the nucleon and $\varepsilon$ – the energy of state.

Thus, at a given temperature T and total concentration n of the mixture, the volume fractions of all components can be expressed in terms of two unknown volume fractions of atomic hydrogen and nitrogen $s_H$, $s_N$. To find these unknowns two equations are needed for total concentration and for the stoichiometric ratio of hydrogen and nitrogen atoms in all compounds in accordance with the formula of the starting compound:

$$\sum_\alpha s_\alpha = 1; \quad (7)$$

$$\frac{\sum_\alpha H_\alpha s_\alpha}{\sum_\alpha N_\alpha s_\alpha} = \begin{cases} 3, & NH_3 \\ 2, & N_2H_4 \end{cases}, \quad (8)$$

where $H_\alpha$, $N_\alpha$ – the number of hydrogen and nitrogen atoms in the molecule $\alpha = N_{N_\alpha} H_{H_\alpha}$.

It is possible to show with the use of (5) that:

$$\ln\frac{s_\alpha}{s_H^{H_\alpha} s_N^{N_\alpha}} + \delta_\alpha \ln\frac{T^{\frac{3}{2}}}{n} + \frac{\Delta\varepsilon_\alpha}{kT} = \text{const}; \quad (9)$$

$$\frac{1}{s_\alpha}\frac{\partial s_\alpha}{\partial T} = \frac{H_\alpha}{s_H}\frac{\partial s_H}{\partial T} + \frac{N_\alpha}{s_N}\frac{\partial s_N}{\partial T} + \frac{\Delta\varepsilon_\alpha}{kT^2} - \frac{3}{2}\delta_\alpha, \quad (10)$$

$$\frac{1}{s_\alpha}\frac{\partial s_\alpha}{\partial n} = \frac{H_\alpha}{s_H}\frac{\partial s_H}{\partial n} + \frac{N_\alpha}{s_N}\frac{\partial s_N}{\partial n} + \frac{\delta_\alpha}{n}, \quad (11)$$

with $\Delta\varepsilon_\alpha = \varepsilon_\alpha - \left(H_\alpha\varepsilon_H + N_\alpha\varepsilon_N\right)$, $\delta_\alpha = H_\alpha + N_\alpha - 1$.

The initial values for the calculation are the mass flow rate $\dot{m}$, stagnant pressure $P_0$ and temperature $T_0$. The calculation begins with the characterization of the mixture at stagnant pressure and temperature in accordance with the methodology presented before.

Then, as the pressure decreases with a certain step, changes in all other parameters are calculated. In this case, substituting (10) and (11) into conditions (7) and (8) leads to a system of linear algebraic equations for the derivatives of all parameters with respect to pressure.

Reducing of pressure continues until the specified nozzle exit area $S_L$ is reached. With the use of the values of the quantities at the nozzle exit thrust, and specific impulse are determined:

$$F = \dot{m}V_L + P_L S_L, \quad (12)$$

$$I_s = \frac{F}{\dot{m}}. \quad (13)$$

## Results and discussion

Below are the results of calculations at a stagnant pressure at the nozzle inlet $P_0=10^4$ Pa.

The temperature dependences of atomic and molecular hydrogen and nitrogen volume fractions are rep-

resented in Fig.1 and 2. Volume fractions of other components do not exceed $10^{-5}$ value.

It can be noted that at temperatures below 4000 K the synthesis of molecular hydrogen is almost completed.

Table. 1 and 2 represents the values of some geometric performance and operational characteristics of thruster.

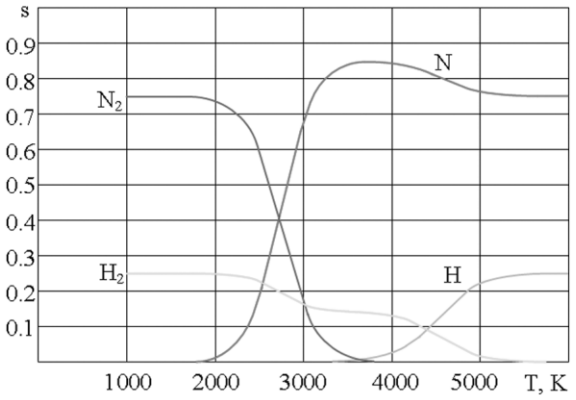


Fig. 1. Volume fractions of $NH_3$ decomposition components at $P=10^4$ Pa

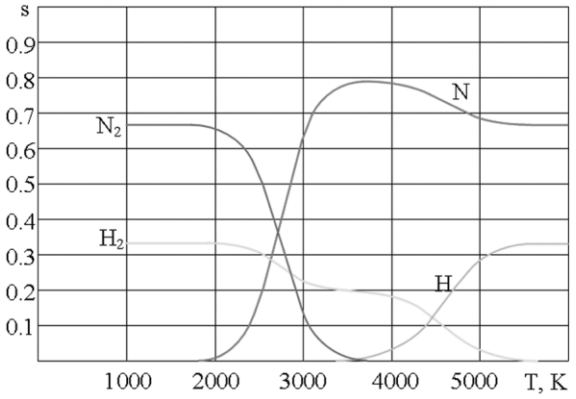


Fig. 2. Volume fractions of $N_2H_4$ decomposition components at $P=10^4$ Pa

Table 1

Parameter distribution when operating on ammonia

| Parameter | Section | | |
|---|---|---|---|
| | entrance | critic | exit |
| P, Pa | $10^4$ | $5.325 \cdot 10^3$ | 9.33 |
| T, K | 4000 | 3524 | 2122 |
| V, m/s | ~0 | 2844 | 7622 |

Table 2

Thruster characteristics when operating on ammonia

| | | |
|---|---|---|
| Discharge chamber radius, mm | $r_d$ | 3.36 |
| Critical section radius, mm | $r_c$ | 1.12 |
| Nozzle cut radius, mm | $r_n$ | 11.2 |
| Thrust, mN | F | 80 |
| Specific impulse, m/s | $I_s$ | 7990 |
| Gas dynamic efficiency | $\eta_g$ | 60.3% |

The distributions of temperature, mass flux velocity, and the achieved value of the specific impulse along the length of the thruster chamber when operating on ammonia are represented in Fig. 3 and 4. The achieved specific impulse value at a certain nozzle cross-section is understood as the value at which this cross-section would be output one.

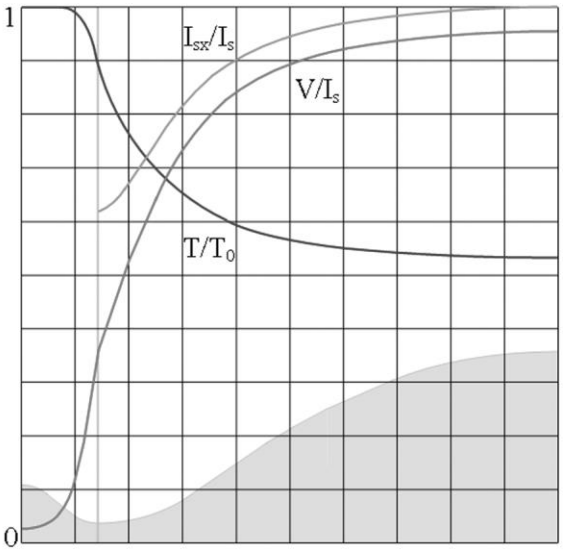


Fig. 3. Distribution of temperature, mass flux velocity, and the achieved value of the specific impulse along the length of the channel when operating on ammonia $T_0=4000$ K, $P_0=10^4$ Pa, $\dot{m}=10$ mg/s

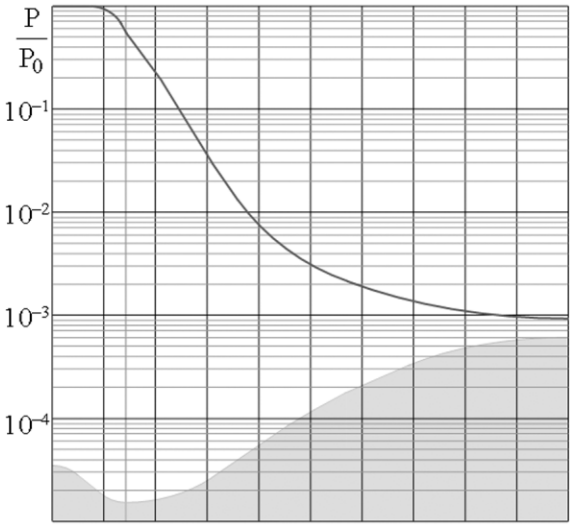


Fig. 4. Pressure distribution along the length of the channel when operating on ammonia $T_0=4000$ K, $P_0=10^4$ Pa, $\dot{m}=10$ mg/s

It can be noted that a decrease in pressure by three orders of magnitude corresponds to a decrease in temperature by less than two times. That is, the process of molecular nitrogen synthesis in its dynamics is close to isothermal phase transitions gas-liquid and liquid-solid.

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A temperature of 4000 K was taken in calculations as the maximum possible in a real design. The calculated specific impulse value is actually equal to what is possible when an RJ operates on hydrogen.

There is no doubt about the possibility of achieving 4000 K in AJ. However, the process before entering the nozzle part of the thruster cannot be considered as equilibrium one and requires the solution of a non-stationary problem, also taking into account the ionized components of the mixture. In this case the time of presence of the propellant in the arc region is almost certainly not enough to dissociate a noticeable part of it.

In RJ with temperatures not higher than 3000 K it is possible to use an extended heater with a sufficiently long stay of the propellant inside. Of course, the specific impulse value will be lower than that shown in Table 2, but still higher than in LPRE

## Conclusions

The results of the work have theoretical significance in modeling processes in ETT when operating on molecular propellant.

The direction of further research is the development of a technique for analyzing the non-stationary dissociation process, also taking into account possible ionization processes of the mixture components.

**Contributions of authors:** conceptualization, methodology – **Serhii Nesterenko;** formulation of tasks, analysis **– Serhii Nesterenko, Peng Shuai;** translation from Chinese **– Peng Shuai;** development of model, software, verification **– Serhii Nesterenko, Peng Shuai;** analysis of results, visualization **– Peng Shuai;** writing – original draft preparation, writing – review and editing **– Peng Shuai.**

All the authors have read and agreed to the published version of this manuscript.

## Conflict of Interest

The authors declare that they have no conflict of interest in relation to this research, whether financial, personal, authorship or otherwise, that could affect the research and its results presented in this paper.

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## Data availability

The manuscript has no associated data.

## Use of Artificial Intelligence

The authors confirm that they did not use artificial intelligence methods while creating the presented work.

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## МОДЕЛЮВАННЯ ПРОЦЕСІВ У СОПЛІ ЕЛЕКТРОНАГРІВНОГО І ЕЛЕКТРОДУГОВОГО ДВИГУНА З МОЛЕКУЛЯРНОЮ РОБОЧОЮ РЕЧОВИНОЮ

***Сергій Нестеренко, Пенг Шуай***

Перший зразок електронагрівного двигуна, який разом з електродуговим двигуном належить до класу електротермічних ракетних двигунів, було розроблено у 1929-33 роках українським конструктором Валентином Глушком. Електротермічні двигуни поєднують газотермодинамічний принцип прискорення у профільованому соплі з використанням електричної енергії для нагрівання робочої речовини. В електронагрівних двигунах енергія надається газу омічним нагрівачем. В електродугових двигунах джерелом енергії слугує дуговий розряд. Перевагу такого типу двигуна вбачали у більшій величині питомого імпульсу порівняно з хімічними ракетними двигунами на твердому або рідкому паливі. З огляду на зворотну пропорційність питомого імпульсу до кореня квадратного з молекулярної маси робочою речовиною спочатку було обрано водень. Однак робота електроракетних двигунів передбачає тривале зберігання робочої речовини і виключає використання водню у рідкому стані. При зберіганні ж у газоподібному стані через значну розрідженість водню маса баку значно перевищує масу самого водню, нівелюючи виграш у зменшенні маси робочої речовини порівняно з хімічними двигунами. Інтерес до розробки електротермічних двигунів відродився завдяки пропозиції використання молекулярних робочих речовин: аміаку і гідразину. В роботі розроблено математичну модель і методику розрахунку термохімічних процесів в первинних реакціях дисоціації і вторинних реакціях синтезу молекулярних азоту і водню в соплі електротермічного двигуна. Методика розрахунку передбачає використання рівнянь нерозривності, імпульсу і енергії з урахуванням зміни молекулярного складу в міру розширення газу в соплі. Проблему браку даних щодо констант реакцій розв'язано за рахунок узагальнення іонізаційної формули Саха на випадок взаємодії атомів азоту і водню. Результати розрахунку демонструють можливість отримання питомого імпульсу на рівні, досягнутому при роботі на водню. Вказано на необхідність використання нестаціонарної форми рівнянь під час створення моделі процесів також у нагрівачі чи дузі двигуна.

**Ключові слова**: електродуговий двигун; електронагрівний двигун; іонізаційна формула Саха; дисоціація; синтез; сопло; аміак; гідразин.

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